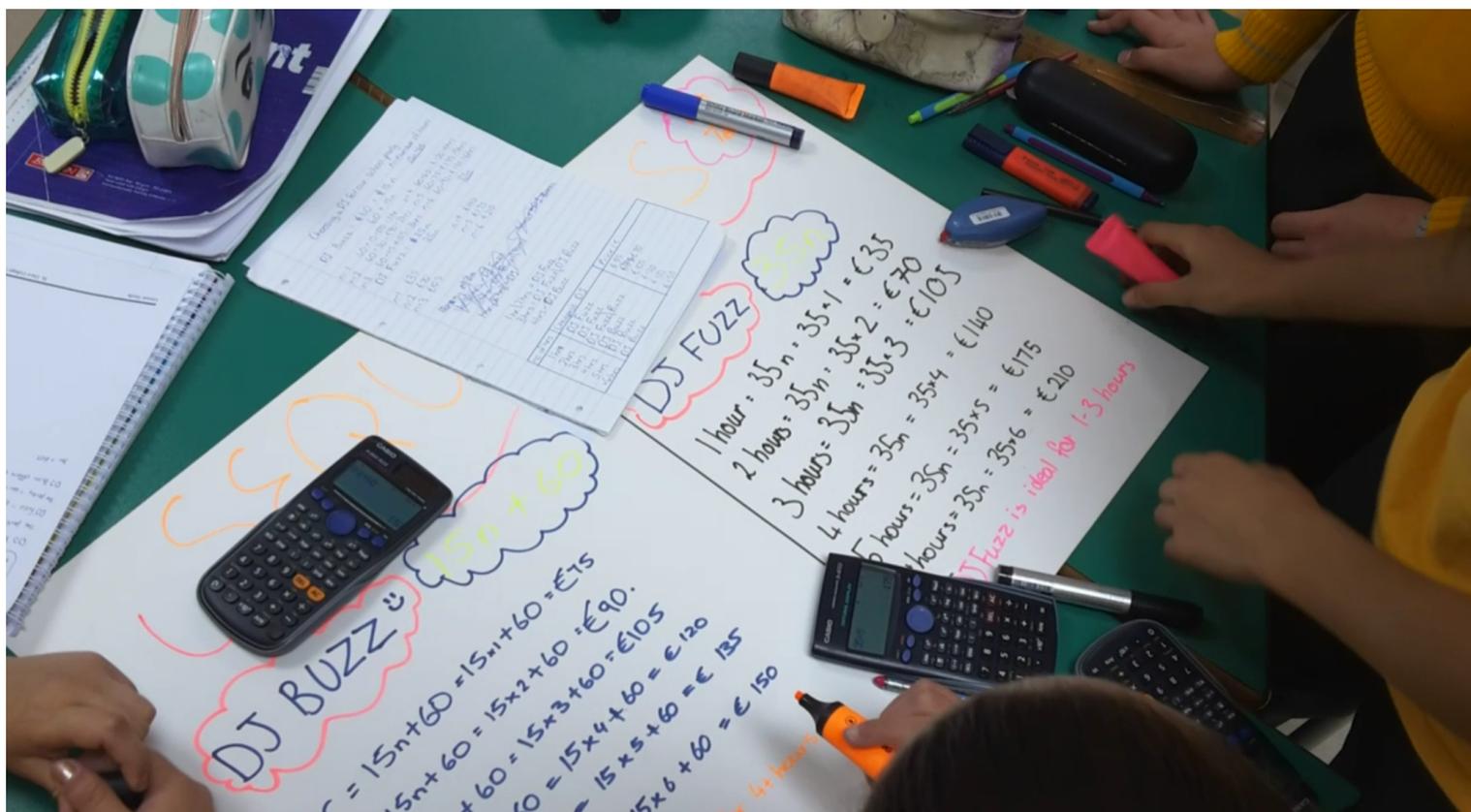


LESSON STUDY REPORT



MAKING MATHEMATICAL CONNECTIONS

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MAKING MATHEMATICAL CONNECTIONS

This report provides a detailed outline of the lesson study process that teachers engaged in to plan, teach, evaluate and re-teach one lesson of mathematics. It includes details about the meetings held, the two lessons taught by two different teachers, the objectives and aims for doing this lesson, the two lesson plans, photographs of the lesson, and comments from students, teachers and observers.

Our aim is to disseminate one practice of teachers working together. Through this document, we seek not only to help others understand the planning and implementation process, but also provide a resource that other teachers may use in their teaching and serving as a support framework for carrying out other lesson studies in whatever subject/area of study.

THE LESSON STUDY TEAM

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ST CLARE COLLEGE
SECONDARY SCHOOL, PEMBROKE (MALTA)
LESSON STUDY CONDUCTED FROM MARCH TO MAY 2017

WHAT IS LESSON STUDY?

Lesson study is a continuing professional development (CPD) model that values teachers as experts. Originating in Japan, lesson study is credited with marked changes in teaching and ongoing teacher development through building professional learning communities (Lewis & Tsuchida, 1997).

Lesson study is a CPD process that engages teachers in reflective and collaborative inquiry (Stigler & Hiebert, 1999). Through lesson study, teachers become deeply engaged in students' thinking through ongoing collaborative lesson planning that aims to create high-quality classroom practices. The foci of lesson study are to:

- *improve the lesson planning process;*
- *refine pedagogical strategies and teaching;*
- *engage teachers in action research-based lessons;*
- *assess student thinking and learning; and*
- *disseminate results*

In this lesson study, a group of teachers and teacher educators/researchers worked together, to identify students' difficulties on a given concept (**Making Mathematical Connections**), studying the related curriculum guidelines, analyzing tasks, and planning what we considered as a suitable lesson to address the proposed concept. This 'research lesson' was taught by a member of the group to a class of students, whereas the others observed the lesson with a focus to collect data on student learning. Similar to action research, this lesson study involved cycles of planning, implementation and evaluation. After teaching the lesson, the group evaluated outcomes using the data collected from observers, students' work and a video recording of the lesson. Finally, the lesson plan was refined and later re-taught by another teacher to another class, with new improvements to the lesson evaluated once again.

WHY LESSON STUDY?

Through lesson study, teachers together with educators collaborate to conduct research on practice and become more knowledgeable about teaching. This kind of CPD enables teachers to create collaborative school structures that facilitate their professional learning and the improvement of student learning. This initiative, the first of its kind locally, aims to address this lacuna through an approach that places practice at the centre of teachers' research dialogue.

THE AIM OF THE LESSON

The aim of this lesson was to help students in ***Making Mathematical Connections*** between topics as they engage in solving a problem. Students were presented with a problem that provided offers by two DJs – DJ Buzz and DJ Fuzz. They were asked to decide which DJ to hire for their school party.

There are many ways in which students could solve the problem – using algebra, substituting values for time, trial and improvement, number machines, sequences, making a table of values and plotting a graph. Our aim was to expose students to all these methods and then encourage them to see the links between each.

As students worked individually, they engaged in using any of these methods. Later, they shared their different methods as they worked in pairs and in small-groups. Our planning of two short whole-class discussions were intended for students to make these links between methods attempted by the different groups.

This lesson had additional goals for students, namely:

- *Engaging in collaborative learning;*
- *Peer tutoring each other;*
- *Presenting, sharing and critiquing ideas;*
- *Undertaking more active learning roles.*

STUDENTS' PRIOR LEARNING

This lesson was planned to take place after Year 9 students following the Track 3 syllabus had knowledge of the following topics:

- *Number machines*
- *Sequences – using and finding the n^{th} term;*
- *Algebra – constructing a formula and substituting values into a formula;*
- *Graphs – constructing a table of values, plotting and interpreting a graph;*
- *Solving simultaneous equations graphically.*

These topics were already covered in class by teachers before this lesson was conducted.

WHY 'MAKING MATHEMATICAL CONNECTIONS' IS IMPORTANT

Too often, teachers tend to present situations in which students focus on remembering procedures or tricks rather than making sense of mathematical ideas. This is, we feel, a constant struggle for teachers because of constraints that the syllabus may provide. However, looking at mathematical content as a connected set of topics and concepts may aid both teachers in teaching the subject and the students in gaining deeper learning.

Taking a mathematical concept and considering how it originates, extends, and connects with other concepts will help students develop a deeper understanding. It is through connections that students may make sense of abstract representations. The connections among concepts and the use of concrete representations certainly led to deeper understanding. We need to seek to plan learning opportunities that ensure that students regularly make connections to help them make sense of the mathematics they are learning.

It is our duty, we feel, to provide such experiences and opportunities that support students in linking their work to mathematical concepts. It is also important for them to make connections between what they are doing and the mathematical ideas that they have learned and those that they are currently working on. This, undoubtedly, takes time and teacher expertise. Hence, our decision to study a lesson based on students making mathematical connections.

In targeting this goal, we sought to enable students to:

- *Recognize and use connections among mathematical topics;*
- *Understand the links between mathematical ideas, topics and concepts;*
- *Identify and apply a range of mathematical skills, strategies and concepts to solve a problem.*

We believe that, when mathematical ideas are connected to each other or to real-world problems, students begin to see mathematics as useful, relevant and linked. Learning mathematics through problems in context and making connections to their prior knowledge may increase students' willingness to engage more deeply with doing and understanding mathematics.

In problem-solving situations, like the one presented in this lesson, students are required to look at their solutions and reflect on possible connections. What students notice in their solution strategy links to current and/or prior learning. Through discussion with their peers and the teacher, students may explore and discover new learning through multiple representations of solution strategies. They may also relate mathematical solutions presented by others to their own strategies, hence generating new mathematical meanings and connections.

MEETING 1 6 th March 2017	<i>Overview of the lesson study process</i>
MEETING 2 13 th March 2017	<i>Identifying possible topics that are challenging to teach</i>
MEETING 3 20 th March 2017	<i>Identifying a problem with student learning that teachers agreed to study</i>
<p><i>An online platform was set up to continue with the discussion in between the weekly meetings. We created a Facebook closed group through which teachers shared ideas, resources and materials, and discussed issues with lesson planning.</i></p> <p><i>This platform was crucial for keeping our momentum in the lesson planning process.</i></p>	
MEETING 4 3 rd April 2017	<i>Refining the learning task</i>
MEETING 5 10 th April 2017	<i>Preparing the lesson plan</i>
MEETING 6 17 th April 2017	<i>Continuing with lesson planning</i>
MEETING 7 3 rd May 2017	<i>Organising the logistics for teaching the first lesson</i>
<p><i>The first lesson trial was done on Monday 8th May 2017.</i></p> <p><i>This lesson was taught during the departmental meeting time-slot because we wanted all teachers involved in the lesson planning to be present. Besides the teacher (James Calleja), there were 12 observers for this lesson: seven teachers of mathematics, an assistant head, a head of department (English) and three members from the Faculty of Education (University of Malta).</i></p>	
MEETING 8 15 th May 2017	<i>Lesson debriefing meeting (all teachers involved in lesson planning were present – see page 19 for more details).</i>
<p><i>The second lesson trial was done on Monday 22nd May 2017.</i></p> <p><i>This lesson was taught during the departmental meeting time-slot because we wanted all teachers involved in the lesson planning to be present. Besides the teacher (Philip Zammit), there were 11 observers for this lesson: six teachers of mathematics, four members from the Faculty of Education (University of Malta) and a mathematics MTL student-teacher.</i></p>	
MEETING 9 22 nd May 2017	<i>Lesson debriefing session (the teacher teaching the lesson, two of the teachers involved in the lesson planning, the four members from the Faculty of Education and the MTL student-teacher were present – see page 31 for more details).</i>

TRIAL 1

LESSON PLANNING STAGE

Planning started off with a brainstorming activity that focused on aspects that teachers would like to address within their planning of the lesson – aspects that they either found challenging to teach and/or difficult for students to understand.

The following ideas were presented:

- *To plan a lesson about constructions because it is one topic that the teacher struggles with when it comes to managing the class with students having different levels of skills in using the compass and constructing angles;*
- *To plan a lesson related to algebra which seems an abstract topic to most students and which usually poses difficulties related to their conceptual understanding;*
- *To plan a lesson on graphs because the topic seems challenging for all year groups. In particular, students do not seem to make a connection between the table of values, the equation of the line and the graph itself.*

Teachers discussed the range of student difficulties that they usually face when teaching the topic of graphs, namely:

- *Drawing of graph and axes*
- *The relationship between the variables x and y*
- *Choosing a suitable scale*
- *Understanding why we use the table of values*
- *Gradient's relation with equation*

Ultimately, following further discussion, the group decided to plan a lesson that provides students with access to mathematical connections between algebra, graphs, sequences and the equation of a line.

The next point we focused on was a potential lesson starter. Again, we brainstormed ideas and teachers suggested that a real-life situation would be ideal for students. We decided to present a scenario where two situations (offers) are presented to students and they need to decide what would be the best option to choose and how they can decide upon it. Organising a birthday party could be a potential task with the teacher presenting two options and asking students to decide which offer to take. Eventually we chose to present a situation of hiring a DJ for our school party. We decided to ask students to consider two offers – one from DJ Buzz and another from DJ Fuzz, and attempt to solve this problem “Which DJ offers the best deal for your party?”.

With this task, we intended to offer an open problem that could be solved in different ways, using different methods. We also wanted students to use graphs at some point during the lesson and to eventually make connections between the different methods used including the graphical solution.

We also designed observation sheets to collect data about student learning.

THE LESSON PLAN

We wanted students to explore multiple ways of solving the DJ problem. Following the teacher presentation of the problem, students were asked to try to solve the problem on their own. Then they were asked to share their solution strategies in pairs before sharing their ideas in a whole-class discussion.

During the whole-class discussion, the teacher would expose students to different ways of solving the problem by moving from substituting values to creating a table of values and then plotting a graph. Once these aspects emerge, the teacher would then ask students to work in groups and design a poster showing two ways for solving the problem. This was eventually followed up by a whole-class presentation of the posters. Eventually, the teacher would conclude the lesson by helping students to see and make connections between the different topics.

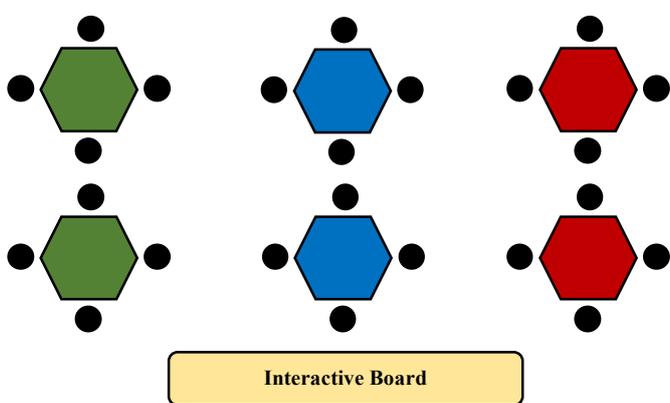
This lesson was video recorded for evaluation purposes.

Lesson title	Making Mathematical Connections
Subject	Mathematics
Teacher	James Calleja
Year group	Year 9 Track 3 – A class of 16 students
Date	8 th May 2017
Time and duration	4 th Lesson: 10:55 – 11:35 (40 minutes)
Lesson purposes	Students will be able to: <ol style="list-style-type: none">1. Relate a real-life problem to mathematics2. Apply strategies for understanding the problem3. Make connections between mathematical topics such as graphs, table of values, algebra, sequences and number machines
Objectives addressed	AL3: Generating terms of a sequence given the formula AL6: Graphs: generating and plotting coordinate pairs that satisfy a linear rule AL4: Solving simultaneous equations graphically: solving problems leading to the solution of simultaneous linear equations
Lesson overview	This lesson links to topics that students have already covered during the year, namely: algebra, straight line graphs, sequences and functions. Students are provided with a real-life problem. They are first asked to think about mathematical topics that are related to the situation/problem offered. We hypothesise that students would suggest the following topics: algebra, simultaneous equations and substitution.

<p>Promoted practices</p>	<p>In planning this lesson, we attempt to incorporate the eight principles of mathematics teaching practices that research indicates need to be consistent components of every mathematics lesson.</p> <table border="1" data-bbox="469 271 1342 889"> <thead> <tr> <th colspan="2" style="background-color: #0056b3; color: white; text-align: center;">Mathematics Teaching Practices</th> </tr> </thead> <tbody> <tr> <td style="background-color: #e6f2ff;">Establish mathematics goals to focus learning.</td> <td>Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</td> </tr> <tr> <td style="background-color: #e6f2ff;">Implement tasks that promote reasoning and problem solving.</td> <td>Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</td> </tr> <tr> <td style="background-color: #e6f2ff;">Use and connect mathematical representations.</td> <td>Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</td> </tr> <tr> <td style="background-color: #e6f2ff;">Facilitate meaningful mathematical discourse.</td> <td>Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</td> </tr> <tr> <td style="background-color: #e6f2ff;">Pose purposeful questions.</td> <td>Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.</td> </tr> <tr> <td style="background-color: #e6f2ff;">Build procedural fluency from conceptual understanding.</td> <td>Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</td> </tr> <tr> <td style="background-color: #e6f2ff;">Support productive struggle in learning mathematics.</td> <td>Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</td> </tr> <tr> <td style="background-color: #e6f2ff;">Elicit and use evidence of student thinking.</td> <td>Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</td> </tr> </tbody> </table> <p style="text-align: center;">Source: NCTM (2014)</p>	Mathematics Teaching Practices		Establish mathematics goals to focus learning.	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<p>This lesson within a unit</p>	<p>This lesson can be taught at two different stages within a unit of study:</p> <ol style="list-style-type: none"> before introducing a set of topics covered in the previous years and for which there is extension material to be covered; or at the end of a unit after a set of topics (straight-line graphs, algebra, sequences, simultaneous equations and functions) have been covered. <p>We decided to take the second approach. Students in this class already covered each of the above topics with their teacher. Our aim is to offer a problem through which students may be able to see and make connections between the different topics when solving the problem.</p>
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<p>Resources</p>	<p>Interactive whiteboard, power-point slides, task worksheets (on A4 paper), graph paper with axes drawn, a chart (for the students' poster presentation), a set of whiteboard markers and a set of cards (for table of values).</p>
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<p>Classroom organisation</p>	<p>Students are seated in groups of 4 around hexagonal tables. For the first part of the lesson (Part 2) they will work on the task on their own. Then they will work in pairs (Part 2 continued) and later in groups of 4 (Part 4).</p> <div style="text-align: center;">  <p style="text-align: center;">Interactive Board</p> </div>
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Part 1
Lesson
introduction

Whole-class
presentation

(3 minutes)

Teacher says:

"Good morning! As you already know, we are here today to study a lesson that a group of teachers planned together. The observers here will be focusing on our planning of the lesson and what you can learn from it. So, try not to worry about them. They are here to help us improve our teaching and your learning of mathematics."

Teacher asks:

"Is this fine with you?"

(Pause of 5 seconds)

"All you need on the table is your pencil case and calculator."

(Pause of 10 seconds)

Teacher continues:

"In this lesson, which is being video recorded, I will present a problem to you. Now the answer to this problem is not important, it is your thinking about solving the problem that I am interested in."

Students are expected to:

Listen attentively to the teacher.

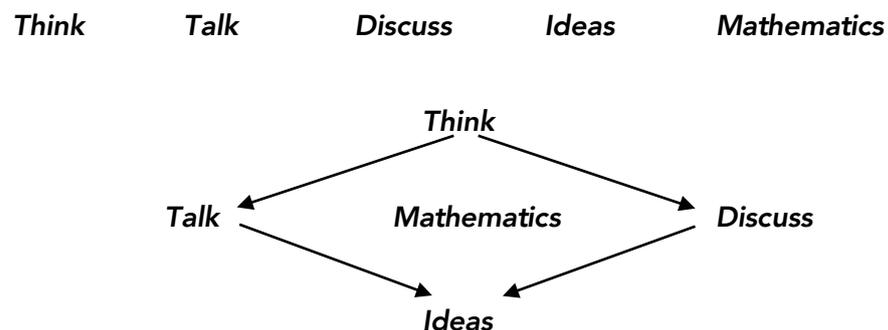
If not, the teacher draws attention to them by saying:

"I need you to listen carefully to what I am saying. This is very important. Please raise your hands if you would like to ask a question."

Teacher continues:

"During the lesson, you will be asked to think and talk about your ideas. You will work on your own first and later as a group. At the end, you will be asked to share your ideas with the whole class. So, your contribution is very important. I would like to hear your ideas so that we can discuss these. The lesson is about seeing how a problem may be tackled in different ways, using different mathematical topics."

Teacher writes on the whiteboard:



Teacher asks:

"Is everything clear so far? Would you like to ask me any questions?"

Part 2 Task presentation	Presenting the Learning Task (1 minute)
Individual work & Pair work (7 minutes)	<p>The Learning Task</p> <p style="text-align: center;">CHOOSING A DJ FOR OUR SCHOOL PARTY</p> <p>You are going to organise an end of school party and want to hire the service of a DJ.</p> <p>You obtained two different quotations for this service.</p> <p>DJ Buzz offers his services by charging €15 per hour plus €60 for the installation of his DJ equipment.</p> <p>DJ Fuzz offers his services by charging €35 per hour.</p> <p>Which DJ offers the best deal for your party?</p>
	<p>The teacher projects task on the interactive board and reads it out.</p> <p>Teacher says: "Here is a situation for you (teacher reads it). You have a few seconds to read it again on your own."</p> <p>The teacher allows about 10 seconds for students to read the problem again and think. Then, the teacher provides a worksheet to each student.</p>
	<p>Individual work (3 minutes)</p> <p>Teacher says: "How would you solve this problem? You have three minutes to work on your own on this. Write your ideas on the worksheet."</p> <p>During this period, the teacher does not intervene but encourages students to try to understand and make an attempt at solving the problem.</p>
	<p>Pair work (3 minutes)</p> <p>Teacher allows three minutes and then asks students to share their ideas with the one next to them.</p> <p>"Now share your ideas with the one next to you. Explain to them how you tried to solve the problem. You might see that you tried to solve the problem differently. That is what we are interested in."</p> <p>Students are expected to discuss: trial and error, trial and improvement or by substituting values of time for each DJ. Yet, others might use algebra and simultaneous equations.</p> <p>The teacher here observes and listens; then later intervenes to uncover students' thinking and reasoning.</p> <p>"How did you come up with that method?"</p> <p>"What do those values represent?"</p>

Addressing student struggles during pair work

Teacher asks:

"What mathematical topics may be related to this problem?"

Students are expected to mention graphs, algebra and proportion.

If students struggle to mention topics or they just have a guess, the teacher will ask the following questions:

- What is known/given?
- What is unknown and would be helpful to know?

Teacher asks:

"Can you tell me what you are given in the problem?"

Students are expected to mention cost per hour and cost for installation.

Teacher asks:

"Can you tell me what is unknown and would be helpful to know?"

Students are expected to mention the **time** for hiring a DJ.

Teacher might also ask:

"Can you now think of mathematical topics related to what you mentioned?"

Students are expected to mention graphs, algebra, sequences and table of values.

Part 3 Main activity

Whole-class
discussion

(9 minutes)

The teacher asks students that they will now be asked to share their ideas and methods of trying to solve the problem.

Teacher says:

"What I am interested in is the different ways that you tried to use to solve this problem. So, now I will be asking you to share your ideas to the whole class."

The teacher now invites some students to share their work, but also call out students to share their ideas if their method was distinctive from the rest.

Students might come up with some of the following methods.

Method 1: Using substitution

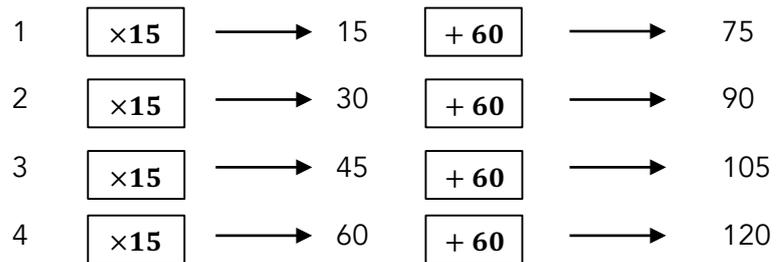
Students might substitute some values of time for both DJs.

	DJ Buzz	DJ Fuzz
Time 1 h:	$15 \times 1 + 60 = \text{€}75$	$35 \times 1 = \text{€}35$
Time 2 h:	$15 \times 2 + 60 = \text{€}90$	$35 \times 2 = \text{€}70$
Time 3 h:	$15 \times 3 + 60 = \text{€}105$	$35 \times 3 = \text{€}105$
Time 4 h:	$15 \times 4 + 60 = \text{€}120$	$35 \times 4 = \text{€}140$
Time 5 h:	$15 \times 5 + 60 = \text{€}135$	$35 \times 5 = \text{€}175$

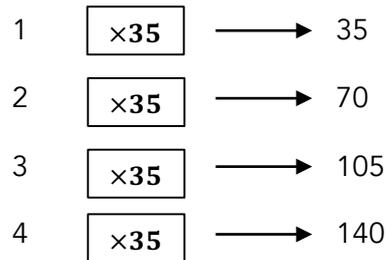
Method 2: Using number/function machines

Students might opt to use a number machine to obtain values of costs.

DJ Buzz



DJ Fuzz



Method 3: Using sequences

Students might represent offers as a sequence.

DJ Buzz: 75, 90, 105, 120, 135, ...

DJ Fuzz: 35, 70, 105, 140, 175, ...

Method 4: Using table of values

Students might opt to draw table of values for both DJs.

For DJ Buzz

Time (h)	1	2	3	4	5	6
Cost (€)	75	90	105	120	135	150

For DJ Fuzz

Time (h)	1	2	3	4	5	6
Cost (€)	35	70	105	140	175	210

Method 5: Using algebra

Students might opt to create two equations.

For DJ Buzz: $C = 15t + 60$

For DJ Fuzz: $C = 35t$

Method 6: Using simultaneous equations

Students might attempt to solve the two equations simultaneously.

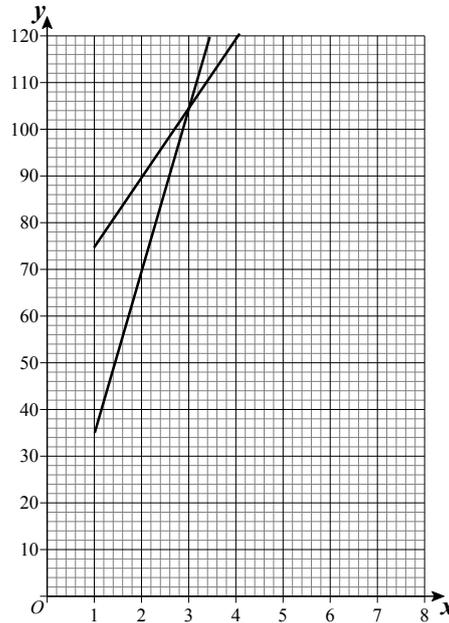
$$35t = 15t + 60$$

$$20t = 60$$

$$t = 3$$

Method 7: Solving using graphs

Students might attempt to solve the problem using graphs. However, we consider this to be very unlikely. Nevertheless, it is important to have graph paper, with given scale on both axes, ready to provide to students should they opt for such a method.



The teacher writes the methods, that students present, on the board.

Substitution	Function Machine	Table of Values	Sequence
Algebra	Simultaneous Equations	Graphs	

Should students not come up with the table of values or find it difficult to relate the values for the cost that each DJ charges for every hour of service to creating a table, the teacher uses the resource cards to construct the table of values with the students.

DJ Buzz

Time	1	2	3	4	5
Cost	75	90	105	120	135

DJ Fuzz

Time	1	2	3	4	5
Cost	35	70	105	140	175

Part 4
Group work

Poster
preparation

(8 minutes)

The teacher asks students to work in groups of four (with their peers on the table) and design a poster explaining, in detail, two methods to show the offers of the two DJs.

Teacher says:

"You will now work as a group. On the chart provided you need to design a poster showing the offers provided by the two DJs using two different representations. You have 8 minutes to do this. Then you will be asked to explain your poster presentation to the whole class."

During this phase, the teacher goes around the groups. The teacher observes and listens first, then intervenes to assess student learning. Possible questions that the teacher may ask:

"How can you tell the difference between the DJs' offers?"

"Which offer would you take and why?"

"How are your methods similar or different?"

"Which of the two methods would you prefer to use?"

Part 4
**Student
presentation**

Whole-class
discussion

(10 minutes)

The teacher says:

"Now I will ask each group to come out and briefly explain the two ways you chose to represent the offers of the two DJs."

The teacher selects the first group to present their poster. Following the presentation, the teacher asks:

"Is there anyone who can add anything to this?"

If none of the groups add anything, the teacher asks:

"Is there any group who had a method similar to these two?"

The teacher then invites the second group to present. This continues until all groups have had an opportunity to present their poster.

Part 5
Teacher
summary

(3 minutes)

Using the power-point presentation, the teacher shows the possible methods to explain the DJs' offers and then shows the connection between each one.

USING SUBSTITUTION

	DJ Buzz	DJ Fuzz
Time 1 h:	$15 \times 1 + 60 = \text{€}75$	$35 \times 1 = \text{€}35$
Time 2 h:	$15 \times 2 + 60 = \text{€}90$	$35 \times 2 = \text{€}70$
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USING NUMBER MACHINES

	DJ Buzz	DJ Fuzz
1	$15 \times 15 \rightarrow 15 \rightarrow +60 \rightarrow 75$	$1 \times 35 \rightarrow 35$
2	$15 \times 15 \rightarrow 30 \rightarrow +60 \rightarrow 90$	$2 \times 35 \rightarrow 70$
3	$15 \times 15 \rightarrow 45 \rightarrow +60 \rightarrow 105$	$3 \times 35 \rightarrow 105$
4	$15 \times 15 \rightarrow 60 \rightarrow +60 \rightarrow 120$	$4 \times 35 \rightarrow 140$

USING A SEQUENCE

DJ Buzz: 75, 90, 105, 120, 135, 150, ... (+15)

DJ Fuzz: 35, 70, 105, 140, 175, 210, ... (+35)

USING A TABLE OF VALUES

DJ Buzz		1	2	3	4	5	6
Time (h)	Cost (€)	75	90	105	120	135	150

DJ Fuzz		1	2	3	4	5	6
Time (h)	Cost (€)	35	70	105	140	175	210

USING ALGEBRA & SIMULTANEOUS EQUATIONS

DJ Buzz

$C = 15t + 60$

DJ Fuzz

$C = 35t$

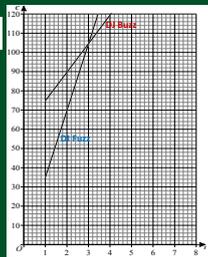
$35t = 15t + 60$

$35t - 15t = 60$

$20t = 60$

$t = 3$

USING GRAPHS

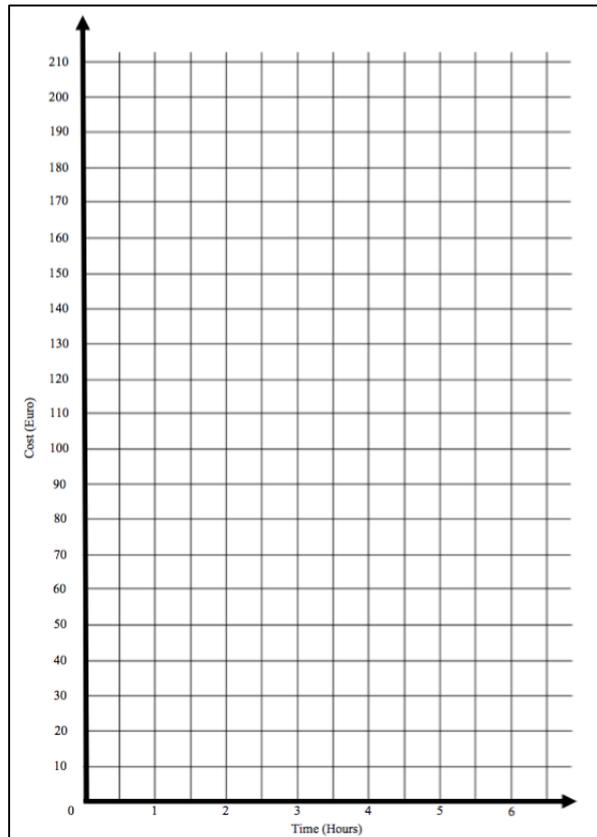


Learning Task Worksheet

Graph Paper

Choosing a DJ

You are going to organise an end of school party and want to hire the service of a DJ. You obtained two different quotations for this service. DJ Buzz offers his services by charging €15 per hour plus €60 for the installation of his DJ equipment. DJ Fuzz offers his services by charging €35 per hour. Which DJ offers the best deal for your party?

OBSERVER EVALUATION SHEET 1

Looking for evidence

What do students do? What questions do they ask? How do students respond to the questions posed? What do they discuss? What do students write? How do they react? What do students struggle with?

Part 1: Individual/Pair work			
Students ...	Yes	No	How is it evident?
Understand their work on the task			
Offer mathematical topics linked to the task			
Contribute ideas based on the problem posed			
Part 2: Whole-class discussion			
Offer ideas about ways to solving the problem			
Show understanding of a valid method to use			
Part 3: Group work			
Contribute ideas to the group			
Challenge what is offered by members of the group			
Explain their thinking when solving task			

OBSERVER EVALUATION SHEET 2

Looking for evidence

What do students do? What questions do they ask? How do students respond to the questions posed? What do they discuss? What do students write? How do they react? What do students struggle with?

Students ...	Yes	No	How is it evident?
Part 4: Whole class presentation			
Present their ideas clearly to the whole-class			
Question the work presented by others			
Make connections between the methods presented			
Part 5: Teacher summary of the main points			
Understand that task links to different topics			
Draw conclusions from the summary presented			
Make other inferences from the two graphs			

Thank you for your feedback!

*The Lesson Study Mathematics Team
St Clare College Secondary School, Pembroke*

LESSON DEBRIEFING MEETING

A number of issues emerged from the data collected through the observation sheets and the video recording of the lesson.

(a) **Positive Aspects:**

- Learning task engaged students in thinking;
- The DJ problem was pitched at the right level offering an achievable cognitive challenge;
- Timer displayed on the interactive board was effective in supporting teacher and students to keep with the time set during each phase of the lesson;
- Pair work was an effective pedagogical approach to use and this helped students in learning from one another;
- Students worked well in groups;
- The whole-class discussion supported students in creating links from their solution strategy (using substitution) to identifying a pattern, using a table of values, drawing a graph and coming up with an equation.

(b) **Areas for Improvement:**

- Introductory part involved too much information for students to be able to make sense of what the lesson involved;
- Students did not have enough time to design the poster;
- Cards used to introduce table of values were too small;
- Students seemed too shy to present their work to the whole-class;
- Prepared power-point slides showing the different solution strategies were not used;
- Connections between topics was not clearly made;
- Lesson closure was rushed and there was no evidence that students made the intended links.

(c) **Amendments to lesson:**

- Students need to be introduced to the structure of the lesson. Hence a chart showing the lesson structure will be prepared to support the teacher in explaining each lesson phase;
- Rather than preparing a poster with two solution strategies, students will be asked to present just one strategy of their choice;
- Solution strategies will be assigned by teacher following a short discussion with the students;
- Teacher will assign 1 minute for each group to present their poster;
- When concluding the lesson, the teacher will use the power-point slides;
- Teacher will focus on the last slide to show the mathematical connections related to solving the problem.

TRIAL 2

THE REVISED LESSON PLAN

In this second trial, we intended to make improvements making sure that students focus on just one solution strategy to present on their poster, thus managing to finish it in the time available. Moreover, we wanted students to have more time available for the final presentation and discussion than they did in the first trial. Again, we intended students to engage in discussion and have opportunities to share ideas and support each other in the learning process. Hence, pair work, group work and class discussions remained key phases within the lesson.

We sought that students could see and make links between the different topics presented and discussed during the lesson. Indeed, the final power-point slide (see page 26) could turn out to be an important tool to address this issue.

This lesson was also video recorded.

Lesson title	Making Mathematical Connections
Subject	Mathematics
Teacher	Philip Zammit
Year group	Year 9 Track 3 – A class of 23 students
Date	22 nd May 2017
Time and duration	4 th Lesson: 10:55 – 11:35 (40 minutes)
Lesson purposes	Students will be able to: <ol style="list-style-type: none">1. Relate a real-life problem to mathematics2. Apply strategies for understanding the problem3. Make connections between mathematical topics such as graphs, table of values, algebra, sequences and number machines
Objectives addressed	AL3: Generating terms of a sequence given the formula AL6: Graphs: generating and plotting coordinate pairs that satisfy a linear rule AL4: Solving simultaneous equations graphically: solving problems leading to the solution of simultaneous linear equations
Lesson overview	This lesson links to topics that students have already covered during the year, namely: algebra, straight line graphs, sequences and functions. Students are provided with a real-life problem. They are first asked to think about mathematical topics that are related to the situation/problem offered. We hypothesise that students would suggest the following topics: algebra, simultaneous equations and substitution.

Promoted practices

In planning this lesson, we attempt to incorporate the eight principles of mathematics teaching practices that research indicates need to be consistent components of every mathematics lesson.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Source: NCTM (2014)

This lesson within a unit

This lesson can be taught at two different stages within a unit of study:

1. before introducing a set of topics covered in the previous years and for which there is extension material to be covered; or
2. at the end of a unit after a set of topics (straight-line graphs, algebra, sequences, simultaneous equations and functions) have been covered.

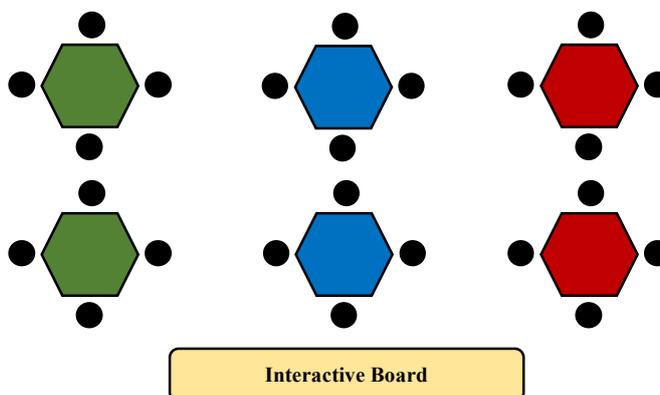
We decided to take the second approach. Students in this class already covered each of the above topics with their teacher. Our aim is to offer a problem through which students may be able to see and make connections between the different topics when using them to solve the problem.

Resources

Interactive whiteboard, power-point slides, task worksheets (on A4 paper), graph paper with axes drawn, a chart (for the students' poster presentation), a set of whiteboard markers and a set of cards (for table of values).

Classroom organisation

Students are seated in groups of 4 around hexagonal tables. For the first part of the lesson (Part 2) they will work on the task on their own. Then they will work in pairs (Part 2) and later in groups of 4 (Part 4).



Part 1
Lesson
introduction

Whole-class
presentation

(3 minutes)

Teacher says:

"Good morning! As you already know, we are here today to study a lesson that a group of teachers planned together. The observers here will be focusing on our planning of the lesson and what you can learn from it. So, try not to worry about them. They are here to help us improve our teaching and your learning of mathematics."

Teacher asks:

"Is this fine with you?"

(Pause of 5 seconds)

"All you need on the table is your pencil case and calculator."

(Pause of 10 seconds)

Teacher continues:

"In this lesson, I will present a problem to you. Now the answer to this problem is not important, it is your thinking about solving the problem that I am interested in."

Students are expected to:

Listen attentively to the teacher.

If not, the teacher draws attention to them by saying:

"I need you to listen carefully to what I am saying. This is very important. Please raise your hands if you would like to ask a question."

Teacher continues:

"The lesson is about seeing how a problem may be tackled in different ways, using different mathematical topics. You will be presented with a task and then asked to work individually. You will then work in pairs to discuss your ideas which we will share afterwards as a whole-class. We will then work in groups of four to present a poster, and finally we will discuss the poster presentations."

Teacher writes on the whiteboard:

INDIVIDUAL WORK	3 minutes
PAIR WORK	3 minutes
WHOLE-CLASS DISCUSSION	10 minutes
GROUP WORK (IN GROUPS OF 4)	8 minutes
WHOLE-CLASS PRESENTATION AND DISCUSSION	15 minutes

Teacher asks:

"Is everything clear so far? Would you like to ask me any questions?"

Part 2 Task presentation	Presenting the Learning Task (1 minute) The Learning Task
Individual work & Pair work (7 minutes)	<div style="text-align: center; background-color: #004d00; color: white; padding: 10px;"> <h2 style="margin: 0;">CHOOSING A DJ FOR OUR SCHOOL PARTY</h2> <p style="margin: 10px 0 0 0;">You are going to organise an end of school party and want to hire the service of a DJ.</p> <p style="margin: 10px 0 0 0;">You obtained two different quotations for this service.</p> <p style="margin: 10px 0 0 0;">DJ Buzz offers his services by charging €15 per hour plus €60 for the installation of his DJ equipment.</p> <p style="margin: 10px 0 0 0;">DJ Fuzz offers his services by charging €35 per hour.</p> <p style="margin: 10px 0 0 0;">Which DJ offers the best deal for your party?</p> </div>
	<p>The teacher projects task on the interactive board and reads it out.</p> <p>Teacher says: "Here is a situation for you (teacher reads it). You have a few seconds to read it again on your own."</p> <p>The teacher allows about 10 seconds for students to read the problem again and think. Then, the teacher provides a worksheet to each student.</p>
	<div style="background-color: #d3d3d3; padding: 5px;"> Individual work (3 minutes) </div> <p>Teacher says: "How would you solve this problem? You have three minutes to work on your own on this. Write your ideas on the worksheet."</p> <p>During this period, the teacher does not intervene but encourages students to try to understand and make an attempt at solving the problem.</p>
	<div style="background-color: #d3d3d3; padding: 5px;"> Pair work (3 minutes) </div> <p>Teacher allows three minutes and then asks students to share their ideas with the one next to them.</p> <p>"Now share your ideas with the one next to you. Explain to them how you tried to solve the problem. You might see that you tried to solve the problem differently. That is what we are interested in."</p> <p>Students are expected to discuss: trial and error, trial and improvement or by substituting values of time for each DJ. Yet, others might use algebra and simultaneous equations.</p> <p>The teacher here observes and listens; then later intervenes to uncover students' thinking and reasoning.</p> <p>"How did you come up with that method?"</p> <p>"What do those values represent?"</p>

Addressing student struggles during pair work

Teacher asks:

"What mathematical topics may be related to this problem?"

Students are expected to mention graphs, algebra and proportion.

If students struggle to mention topics or they just have a guess, the teacher will ask the following questions:

- What is known/given?
- What is unknown and would be helpful to know?

Teacher asks:

"Can you tell me what you are given in the problem?"

Students are expected to mention the cost per hour and the cost for installation.

Teacher asks:

"Can you tell me what is unknown and would be helpful to know?"

Students are expected to mention the **time** for hiring a DJ.

Teacher might also ask:

"Can you now think of mathematical topics related to what you mentioned?"

Students are expected to mention graphs, algebra, sequences and table of values.

Part 3 Main activity

Whole-class
discussion

(8 minutes)

The teacher asks students that they will now be asked to share their ideas and methods of trying to solve the problem.

Teacher says:

"What I am interested in is the different ways that you tried to use to solve this problem. So, now I will be asking you to share your ideas to the whole class."

The teacher now invites some students to share their work, but also call out students to share their ideas if their method was distinctive from the rest.

Students might come up with some of the following methods.

Method 1: Using substitution

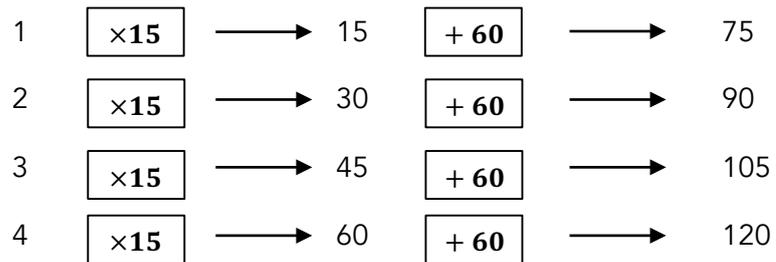
Students might substitute some values of time for both DJs.

	DJ Buzz	DJ Fuzz
Time 1 h:	$15 \times 1 + 60 = \text{€}75$	$35 \times 1 = \text{€}35$
Time 2 h:	$15 \times 2 + 60 = \text{€}90$	$35 \times 2 = \text{€}70$
Time 3 h:	$15 \times 3 + 60 = \text{€}105$	$35 \times 3 = \text{€}105$
Time 4 h:	$15 \times 4 + 60 = \text{€}120$	$35 \times 4 = \text{€}140$
Time 5 h:	$15 \times 5 + 60 = \text{€}135$	$35 \times 5 = \text{€}175$

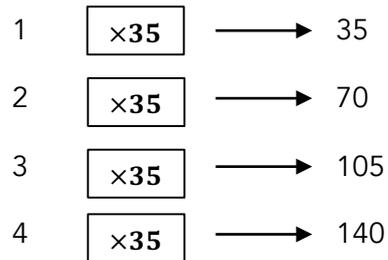
Method 2: Using number/function machines

Students might opt to use a number machine to obtain values of costs.

DJ Buzz



DJ Fuzz



Method 3: Using sequences

Students might represent offers as a sequence.

DJ Buzz: 75, 90, 105, 120, 135, ...

DJ Fuzz: 35, 70, 105, 140, 175, ...

Method 4: Using table of values

Students might opt to draw table of values for both DJs.

For DJ Buzz

Time (h)	1	2	3	4	5	6
Cost (€)	75	90	105	120	135	150

For DJ Fuzz

Time (h)	1	2	3	4	5	6
Cost (€)	35	70	105	140	175	210

Method 5: Using algebra

Students might opt to create two equations.

For DJ Buzz: $C = 15t + 60$

For DJ Fuzz: $C = 35t$

Method 6: Using simultaneous equations

Students might attempt to solve the two equations simultaneously.

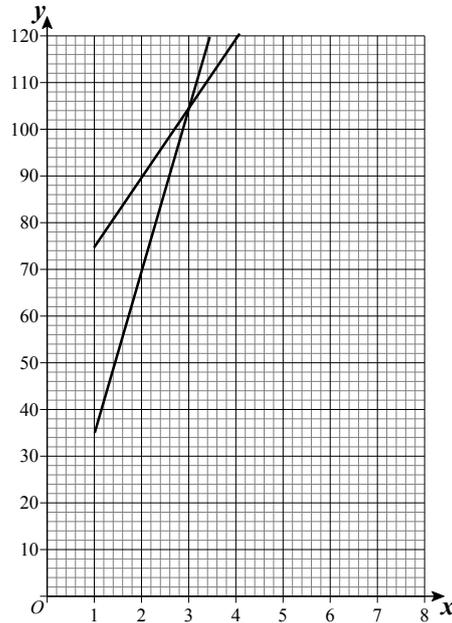
$$35t = 15t + 60$$

$$20t = 60$$

$$t = 3$$

Method 7: Solving using graphs

Students might attempt to solve the problem using graphs. However, we consider this to be very unlikely. Nevertheless, it is important to have graph paper, with given scale on both axes, ready to provide to students should they opt for such a method.



The teacher writes the methods, that students present, on the board.

Substitution **Function Machine** **Table of Values** **Sequence**
Algebra **Simultaneous Equations** **Graphs**

Should students not come up with the table of values or find it difficult to relate the values for the cost that each DJ charges for every hour of service to creating a table, the teacher uses the resource cards to construct the table of values with the students.

DJ Buzz

Time	1	2	3	4	5
Cost	75	90	105	120	135

DJ Fuzz

Time	1	2	3	4	5
Cost	35	70	105	140	175

Part 4
Group work

Poster
preparation

(8 minutes)

The teacher asks students to work in groups of four (with their peers on the table) and design a poster explaining, in detail, one method to show the offers of the two DJs.

Whiteboard

Substitution	Sequence/Pattern	
Table of Values	Equation	Graph

Teacher says:

"You will now work as a group. On the chart provided you need to design a poster showing the offers provided by the two DJs using one particular representation. You have 8 minutes to do this. Then you will be asked to explain your poster presentation to the whole class."

Teacher continues:

"There are five different ways to represent the problem. Which group would like to present the substitution method? Who will work on presenting the offers as sequences or patterns of numbers? Which group will choose the tables of values? I need one group to show us the equations. Who will take this? And who will show us the offers on a graph?"

After selecting a method, students work on the poster presentation comparing the two DJ offers presented in the task.

During this phase, the teacher goes around the groups. The teacher observes and listens first, then intervenes to assess student learning. Possible questions that the teacher may ask:

"How can you tell the difference between the DJs' offers?"

"Which offer would you take and why?"

"How are your methods similar or different?"

"Which of the two methods would you prefer to use?"

Part 4
**Student
presentation**

Whole-class
discussion

(11 minutes)

The teacher says:

"Now I will ask each group to come out and briefly explain the method you chose to represent the offers of the two DJs."

The teacher selects the first group to present their poster. Following the presentation, the teacher asks:

"Is there anyone who can add anything to this?"

If none of the groups add anything, the teacher asks:

"Is there any group who had a method similar to these two?"

The teacher then invites the second group to present. This continues until all groups have had an opportunity to present their poster.

Part 5
Teacher
summary

(3 minutes)

Using the power-point presentation, the teacher shows the possible methods to explain the DJs' offers and then shows the connection between each one.

USING SUBSTITUTION		USING NUMBER MACHINES																													
DJ Buzz	DJ Fuzz	DJ Buzz	DJ Fuzz																												
Time 1 h: $15 \times 1 + 60 = \text{€}75$	$35 \times 1 = \text{€}35$	1 $\times 15 \rightarrow 15$ $+ 60 \rightarrow 75$	1 $\times 35 \rightarrow 35$																												
Time 2 h: $15 \times 2 + 60 = \text{€}90$	$35 \times 2 = \text{€}70$	2 $\times 15 \rightarrow 30$ $+ 60 \rightarrow 90$	2 $\times 35 \rightarrow 70$																												
Time 3 h: $15 \times 3 + 60 = \text{€}105$	$35 \times 3 = \text{€}105$	3 $\times 15 \rightarrow 45$ $+ 60 \rightarrow 105$	3 $\times 35 \rightarrow 105$																												
Time 4 h: $15 \times 4 + 60 = \text{€}120$	$35 \times 4 = \text{€}140$	4 $\times 15 \rightarrow 60$ $+ 60 \rightarrow 120$	4 $\times 35 \rightarrow 140$																												
Time 5 h: $15 \times 5 + 60 = \text{€}135$	$35 \times 5 = \text{€}175$																														
USING A SEQUENCE		USING A TABLE OF VALUES																													
DJ Buzz	DJ Fuzz	DJ Buzz	DJ Fuzz																												
75, 90, 105, 120, 135, 150, ...	35, 70, 105, 140, 175, 210, ...	<table border="1" style="font-size: small;"> <tr><th>Time (h)</th><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr> <tr><th>Cost (€)</th><td>75</td><td>90</td><td>105</td><td>120</td><td>135</td><td>150</td></tr> </table>	Time (h)	1	2	3	4	5	6	Cost (€)	75	90	105	120	135	150	<table border="1" style="font-size: small;"> <tr><th>Time (h)</th><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr> <tr><th>Cost (€)</th><td>35</td><td>70</td><td>105</td><td>140</td><td>175</td><td>210</td></tr> </table>	Time (h)	1	2	3	4	5	6	Cost (€)	35	70	105	140	175	210
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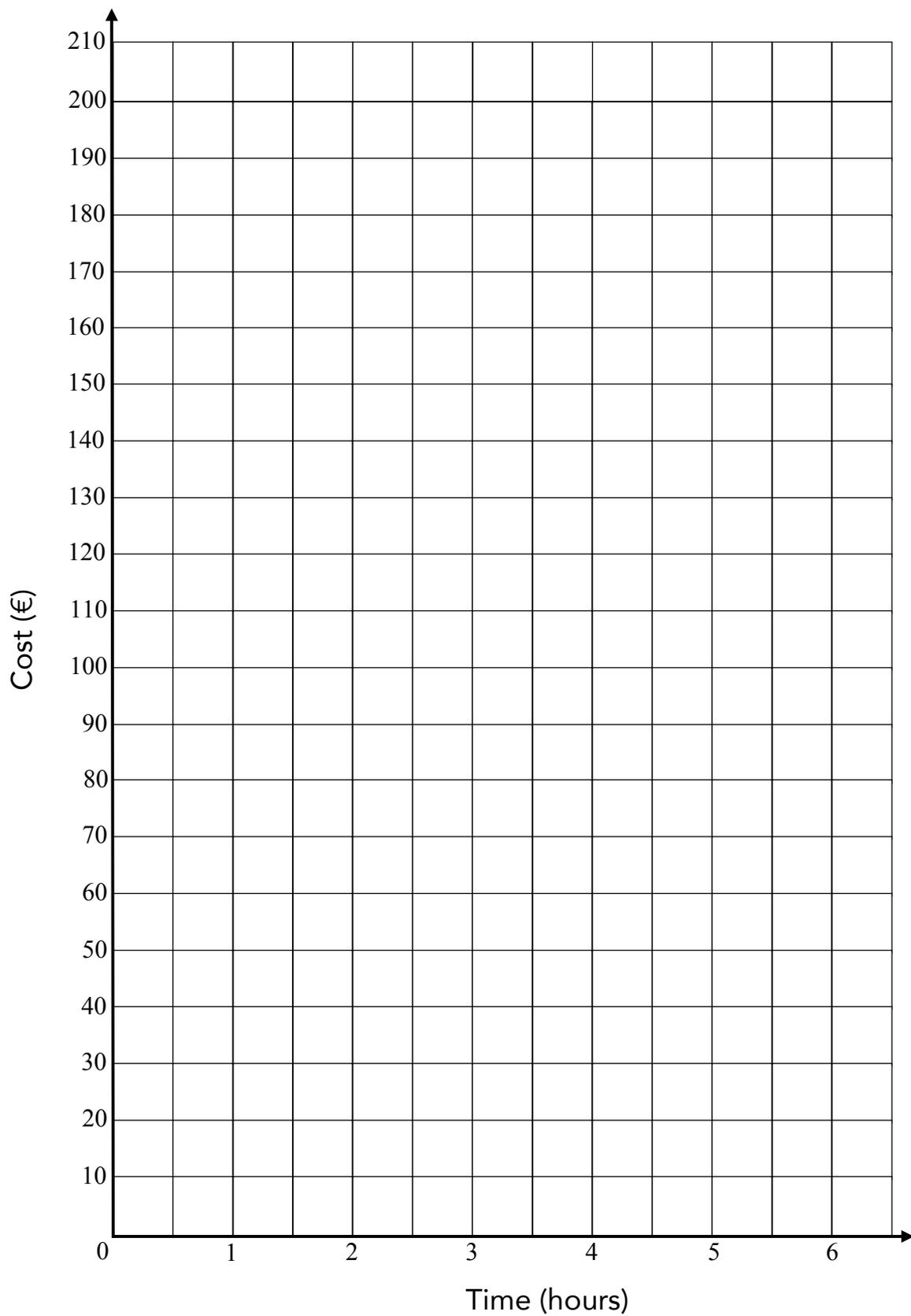
LEARNING TASK WORKSHEET

Choosing a DJ

You are going to organise an end of school party and want to hire the service of a DJ. You obtained two different quotations for this service. DJ Buzz offers his services by charging €15 per hour plus €60 for the installation of his DJ equipment. DJ Fuzz offers his services by charging €35 per hour. Which DJ offers the best deal for your party?



GRAPH PAPER



LESSON DEBRIEFING SESSION

A number of issues emerged from the data collected through the feedback provided by our eleven observers.

(a) **Positive aspects experienced by the teacher**

- Students understood the problem and all managed to come up with a solution to the problem
- The structure of the lesson offered students different learning opportunities – individually, in pairs, as a small group and through the whole-class presentation and discussion
- There was also use of a variety of resources – task handout, graph, poster, and interactive board slides

(b) **Positive aspects highlighted by observers**

- Time spent on each activity was adequate and well managed
- Students were confident communicating their ideas with the teacher
- Students did not seem distracted by those observing them
- Students seemed very focused on what they were working on
- Despite the different work involved, the lesson moved smoothly from one phase to another
- The teacher appeared to have a very good relationship with his class – it was evident that he knew his students well
- The teacher looked calm and confident, and this positively influenced student participation
- Teacher elicited ideas from students and did not rush things
- Problem was pitched at the right level for students; it was neither too easy nor too difficult
- Students came up with the solution strategies and their understanding of the concepts discussed was evident
- Students owned the lesson and the learning that was going on
- Hanging posters on a string was successful in exhibiting students' work
- A good learning experience for the students and all teachers involved

(c) **Areas for Improvement:**

- Students did not seem confident in using mathematical terminology while explaining their work. For example, although they used algebra, they did not mention this as one way of solving the problem.
- Students need to be exposed to such skills – collaborative learning, discussion, and presenting in front of the whole-class
- Could such a scripted lesson offer a constraint to potential unpredictable learning opportunities that may arise from students?
- Lesson could have included ground rules that inform students about their roles during group activity and whole-class presentation

STUDENTS' EXPERIENCES

Following each lesson trial, students were asked to write about their experience, what they liked, felt uncomfortable doing and to provide suggestions about things that they would change. All students reported enjoying the lesson study experience. Most of them identified working in groups as something that they liked and enjoyed doing. Below are some of the most common comments.

What I liked about the lesson...

We could give our own opinions
That we worked in groups
The DJ problem was interesting
It was an extra activity and we had fun
The group work allowed us to share ideas and respond to criticism
The lesson was interactive, the teacher wasn't the only one speaking
We all got along well together
The teacher did not tell us what topic he expected us to work out with
The lesson was interactive and we expressed our ideas
We worked as a team and shared our ideas

What I felt uncomfortable with during the lesson...

The camera/filming
We didn't have a lot of time to work out the problem
There were too many people staring at us
We had to go out in front of other students to explain what we did
Lack of explanation before we worked out the problem

Further comments...

I would like to have more group work during the Maths lessons and more Maths activities like these
It was fun and a creative experience
It was a different type of lesson

Three typical student writings about their experience

"I like the way the lesson was organised and how it was taught. I learnt how the different topics can all be used to solve a single problem. I found this lesson more interesting than the usual ones."

"The lesson was fun and I liked it since we worked more in a group. However, I felt pressured since there were a lot of people whom I didn't know and they were observing our work. I learnt that all the topics in Maths add up together and a problem can be solved in different ways."

"This lesson was very nice because of the group work. I learnt that one problem can be solved with a lot of methods."

TEACHERS' REFLECTIONS

Teachers involved provided the following reflections about their participation.

It was a very positive experience, something voluntary and not imposed on us. It was something new, something that I had never experienced before. I had the opportunity to be part of a team that spent some time preparing a special lesson. I had the opportunity to learn a lot from this activity: enriching my pedagogy and mathematical content, and enhancing my communication and collaboration skills. At first I had a lot of worries, such as: where are we going to start from, what does lesson study entail, what is expected of us, and will we be able to reach our targets. But in the end, as a team, we produced something created by all of us; something that belongs to all of us.

Stefan Azzopardi

This lesson study was a very positive experience, beneficial for my professional development as a teacher. Collaborating with other teachers and actually observing teachers teach inspired motivation to try out different techniques and encouraged me to reflect on my own practice as well as think of ways to improve. Planning a lesson together allowed us to deal with a topic that was difficult for us to teach and plan it in an efficient and effective manner. Doing the lesson twice allowed us to reflect on and improve the lesson. I believe this lesson study was truly successful. But, it required a lot of planning time, and I would not have been able to plan this on my own. Since we were a big group, it was not always easy to agree on an approach. However, each teacher's varied ideas and experiences were beneficial to learn from and create a better lesson. My question is: "How can teachers engage in such collaborative practice on a more regular basis?"

Mariette Bartolo

This was a new experience in teaching for me as I got to work hand in hand with many colleagues on a short school project which focused on improving student learning. I had the opportunity to build and develop ideas and receive immediate feedback on the work. I feel that the lesson study was a success not only for the students who took part in it but also for the mathematics team that helped to create such an amazing lesson. It was difficult to put up all the ideas in such a short amount of time but the work we put together will surely help many students as we can make use of several aspects of that lesson and incorporate them into our usual lessons.

Ian Buttigieg

Overall this was a more positive experience than I expected. We broke the lesson down into a lot of detail and analysed each part carefully. Then we did the same after the first trial. It really reminded me of the importance of self-evaluation and how I don't find enough time and make enough effort to do it. One of the biggest challenges for me was timing and how I did not think we could fit everything into one lesson without it feeling rushed and confusing. However, especially by trial 2, we managed to time it perfectly in such a way that the students still got the full learning experience. I think it was one of the most important achievements that would not have been successful if we had not evaluated the first trial and had a second go.

Tamara Azzopardi

This was my second Lesson Study experience. A difference I found was with the fact that during discussion meetings we had observers who gave us their opinions and insights depending on their area of expertise; being mathematics, pedagogy or methodology. During meetings, we had the opportunity to discuss the teaching of mathematics with our colleagues, something which we rarely have time for in our busy schedules. What struck me was how engaged the students were in the lesson and how they enjoyed it while learning about important higher order mathematical connections. As a developmental teaching milestone, this lesson study made me aware of the need of incorporating more structured pair and group work in my lessons while also encouraging students to present their work in front of other students to help them improve in their public speaking skills.

Gabriella Delia Cumbo

This was the first time that I had the opportunity to collaborate with other mathematics teachers and I must say that it was a learning experience for me. Looking back, I see that working together as a team, works to our benefit since different perspectives, ideas and teachers' characteristics are brought together and challenged so as to come up with an ideal lesson. Moreover, the decision of doing a lesson about a mathematical concept that students find it hard to understand helped me think more deeply about the topic, about how students reason out and even how to apply it. Connecting different mathematical concepts together to represent solutions to just one problem was easier to plan with teachers working together rather than planning it on your own. Timing was a challenge since we were a large group of teachers, each one of us with a different idea brought at the table to be discussed. Yet, I must say that evaluating the lesson after the first trial was helpful to make improvements to the lesson.

Jessica Bajada

Being the first lesson study I was actively involved in, I believe it was one of the most positive experiences in my teaching career so far. I had the opportunity to work collaboratively with other professional teachers, something that unfortunately does not occur frequently. Sharing and listening of ideas, from both experienced teachers and newly qualified ones with innovative ideas, inevitably changed my teaching practices and perspectives. I realised that my teaching ideas, lesson plans and methods do not always transform themselves in the best learning experiences for my students. Therefore, the discussions held with other professionals during the lesson study, gave me the chance to enhance my overall teaching pedagogy.

Such study requires hours of planning and discussions. Conducting eight meetings and other discussion time on social media to complete the plan of one lesson is quite time consuming. Having said this, I still believe that the overall benefits of the lesson study exceeds its disadvantages and challenges. We managed to form a strong relationship between the mathematics teacher community, in which we managed to grow professionally as teachers in the attempt to give a better learning experience to our students.

Philip Zammit

OBSERVERS' COMMENTS

Observers were asked to provide their views with regards to lesson study as a professional development experience for teachers.

I had the pleasure to observe two lesson study meetings which included a preparatory meeting and an evaluation of the first trial lesson. It was very encouraging to observe a group of teachers who are willing to commit themselves to improving their practices towards the benefit of their students' understanding. During these sessions, I observed teachers working in a collaborative environment where they shared their knowledge, expertise and experiences. Although lesson study is a lengthy process, it enabled the teachers to gradually develop a critical eye towards the development and delivery of their own lessons. This critical approach was evident in the issues raised during the evaluation discussion that followed the first trial. Through lesson study these mathematics teachers have managed to create a community that is contributing to their own professional growth.

Miriam Bugeja

Education Officer Mathematics Secondary
Ministry for Education and Employment

It was indeed a pleasure to be part of this lesson study exercise. I do not often have the opportunity, and the luxury, to observe a group of teachers working on a very specific task and then enact it. This micro analysis approach helped me to understand particular aspects of the teaching and learning involved in classroom setting, but also the way these teachers worked, as a team, to develop their expertise, share their craft and question each other's practices. I believe that we all learnt from this experience - the students benefitted from a meticulously planned lesson, the teachers increased their knowledge and became more confident in their ideas and strategies, and the observers gained very enriching insights of what goes on in a teacher's mind when planning a lesson, and then delivering it. I highly recommend this exercise to other groups of teachers as I believe it is a very valid form of professional learning.

Michelle Attard Tonna

Department of Leadership for Learning and Innovation
Faculty of Education

It was a pleasure assisting a lesson study and seeing it work in action. The teachers involved were very enthusiastic and professional in their contributions to design an appealing lesson to the students. Having professionals plan a lesson together, jointly observing the teaching and learning taking place and coming back together to discuss its effectiveness, tells a lot on how quality lessons should be developed. On the flip side of things, I believe that, unless there is a well-planned structure for collaborative preparation to take place within schools, such an approach will be difficult to take off and be sustainable. Lesson study offers an innovative way of planning and delivering a lesson. Yet, it necessitates the development of quality schools; schools that embrace the will to share ideas and resources, to discuss and do research and to revisit the curriculum in offering students what they really need to learn.

Mario Testa

Department of Leadership for Learning and Innovation
Faculty of Education

My experience with lesson study has been most positive. I saw teachers collaborating in a most empowering and fruitful way. Not only were they able to identify something to work on, but they were also able to plan a lesson that by and large addresses the concerns that they had previously identified. Another notable thing I noted was how these teachers, contrary to what normally happens, were able to act as professionals in front of a selection of people who attended their meetings and the two trials of the designed lesson. This is most refreshing as it represents teachers as knowledgeable professionals who can act as a community to plan, implement and evaluate. This experience has served to give us an idea how teachers can take CPD into their own hands and develop professionally as a result. This experience has convinced me that it is also important that we take elements of lesson study and consider ways how this can be used with ITE students.

Michael Buhagiar

*Department of Mathematics and Science Education
Faculty of Education*

As I see it lesson study research has two main aims: the researching, planning, teaching, observing, evaluating and refining of a lesson or group of lessons that can serve as examples of good practice to others, and, the building of a school-based community of practice that promotes collaboration, discussion, reflection and the sharing of ideas and resources. From what I saw in my two visits to Pembroke Secondary School both of these aims are being achieved. Although both aims are important, for me, it is the latter which takes precedence because having teachers working as a team will invariably have a positive effect not only on their effectiveness in the classroom but also on their overall professional development and their job satisfaction. It can also serve to reduce teacher stress. It is clear that a true community of practice is slowly being built up at Pembroke Secondary school and, hopefully, this can be carefully encouraged and nurtured in the years to come. The dissemination of the products of this research – lesson plan, accompanying video and short paper – to other schools should be pursued with vigor and tact as it can encourage teachers in those schools to team together and do something similar.

Leonard Bezzina

*Department of Mathematics and Science Education
Faculty of Education*

SOME FINAL REFLECTIONS

As a leader and facilitator of this lesson study group, this was an exciting journey and a positive learning experience for me too. Those reading this report, and keen to start a lesson study group, may benefit from the guidelines I share with you below.

Initiating and sustaining lesson study requires, I believe, fundamental conditions. Primarily, lesson study requires a small group, between three to six teachers, who are willing to work and learn together. Working together necessitates a non-threatening and non-judgmental environment that supports teachers to speak freely about concerns, challenges and dilemmas in their teaching. Professional learning communities flourish when teachers feel safe to share ideas and, as a result, take responsibility and assume ownership over their personal and group learning.

The person leading and facilitating discussions plays a crucial role as well. The facilitator's role is to probe and prompt teachers to think and reflect, by challenging views and insights through purposeful questioning. The facilitator must ensure that all members of the community have opportunities to participate in the group discussions. Having said that, some teachers may take a more leading role than others. There is nothing wrong with this. However, it is important that all members of the community are involved. Teachers should not be told what and how to do things; they are treated as experts, as researchers and as decisions makers.

Collaborative planning is a long process. To engage in lesson study, teachers need to be provided with time to meet on a regular basis. Professional development leaders and providers need to make and plan time for collaboration. SMTs need to seek support from the Directorate of Learning and Assessment Programmes (MEDE) so that teachers are provided with adequate time to meet, share and plan high-quality lessons together. But SMTs also need to get actively involved in lesson study. It is through their active engagement – attending teacher meetings and acting as observers in lesson studies – that SMTs may be in a better position to appreciate the benefits that lesson study offers towards creating and cultivating a collaborative culture. Collaboration is vital and a life-line for schools in supporting the continuing professional development and learning of our teachers and, more importantly, towards improving student learning.

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